THINKING SCHOOLS
ACADEMY TRUST

Mathematics

Name:________________________________

Class:________________________________

'Transforming Life chances'

Children first- Aspire- Challenge- Achieve

Aspire: To be the best I can be in everything that I try to do. To use the adults and resources available both at school and at home, to aspire for personal excellence and professional competence.

Challenge: To aim high, to push my limits to be able to strive for the highest possible achievements. To make every minute count to by maximising all learning opportunities both at school and at home. To seek challenge and to use my thinking tools to develop my thinking and push myself forward. To be responsible and in control of my own destiny. To be a skilled, independent, reflective learner.

Achieve: To demonstrate the highest levels of thinking and habits. To question, to challenge, to think independently and interdependently to achieve my personal academic aims. To be proud of who I am and what I achieve.

'You are who you choose to be!'

	T6	T1	T2	T3	T4	Target

Term 1	
Arithmetic	
Paper 1	
Paper 2	
Total	

Term 2	
Arithmetic	
Paper 1	
Paper 2	
Total	

Term 3	
Arithmetic	
Paper 1	
Paper 2	
Total	

Term 4	
Arithmetic	
Paper 1	
Paper 2	
Total	

Target	
Arithmetic	
Paper 1	
Paper 2	
Total	

Number and Place value

M1a: Can read, write and order whole numbers up to 10,000,000

Place value is the number system that we use to describe the position of each digit within a number.
Whole numbers are numbers that do NOT include fractions and decimals.

Millions	Hundreds of Thousands	Tens of Thousands	Thousands	Hundreds	Tens	Ones
M	**HTh**	**TTh**	**T**	**H**	**T**	**Ø**

We can use this model of place value to help us read, write and order whole numbers up to 10,000,000.
Read **5683**

T	H	T	Ø			
5	**6**	**8**	**3**	=5000+ 600+ 80+ 3	5 thousand, 6 hundred and eighty-three	five thousand, six hundred and eighty-three

Read **467293**- to help read the number count back three digits from the right and place a **comma**. 467,293

HTh	TTh	T	H	T	Ø			
4	**6**	**7**	**2**	**9**	**3**	=400,000+ 60,000+ 7,000+ 200+ 90+ 3	4 hundred and 67 thousand, 2 hundred and 93	Four hundred and sixty-seven thousand, two hundred and ninety-three.

Ordering Whole Numbers-

Note: you should be able to order whole numbers with up to 5 digits.

Let's look at an example with 4 digit numbers.
Put these numbers in ascending order: 4521, 2451, 5124, 2154, 5214
Ascending means smallest to biggest.
Let's look at the thousands column. There are two numbers with 2 thousands: 2451 and 2154. We now need to look at the hundreds column. We can see that 2451 has 4 hundreds but 2154 has only 1 hundred so it is smaller. The next smallest number is 4521. We then have 2 numbers with 5 in the thousands column. 5124 and 5214. We can see that 5214 is bigger as it has 2 hundreds. So, our numbers in ascending order are
2154, 2451, 4521, 5124, 5214
Hint: When answering questions such as this, make sure your final list has all the numbers and none have been left out.

T	H	T	Ø
5	2	1	4
5	1	2	4
4	5	2	1
2	4	5	1
2	1	5	4

Look at numbers as a whole
What is the biggest number? In this case it is a thousand number- therefore you need to think about place value up to Thousands.

Look at numbers in place value order-
Thousands-Hundreds-Tens-Ones

M1b: Can read, write and order numbers up to 3 decimal places

Remember: When working with decimals, it is very important that you know and understand the place value of numbers.

Thousands (1000)	Hundreds (100)	Tens (10)	Ones (1)	●	Tenths ($\frac{1}{10}$)	Hundredths ($\frac{1}{100}$)	Thousandths ($\frac{1}{1000}$)
		3	**2**	**.**	**4**	**5**	**7**

For example, we can use this chart to see that the number 32.457 is made up of:
3 tens, 2 ones, 4 tenths (or 4/10 or 0.4), 5 hundredths (or 5/100 or 0.05) and 7 thousandths (or 7/1000 or 0.007).

This number is read or written in words as thirty two point four five seven.

Read 21.098 **-T**his number is read or written in words as **twenty one point zero nine eight**.

We are sometimes given decimal numbers written in words and we have to read, understand the number and write it in figures. Let's look at some examples:

Write forty nine point zero nine four in figures	This is written in figures as 49.094
Write thirty point two zero eight in figures	**This is written in figures as 30.208**

We will sometimes be asked questions to show that we understand place value. Let's look at an example:

Write this total as a decimal	$4 + \frac{4}{10} + \frac{2}{100} =$	We can see from our place value chart that this number will be 4.42

Write a number in the box to make this correct:

7.645 = 7 + 0.6 + □ + 0.005

Ones	**.**	$\frac{1}{10}$ ***Tenths***	$\frac{1}{100}$ ***Hundredths***	$\frac{1}{1000}$ ***Thousandths***
7	**.**	***0.6***	***0.04***	***0.005***

We know that the number 7.645 is 7 ones, 6 tenths, 4 hundredths and 5 thousandths or

7.645 = 7 + 0.6 + 0.04 + 0.005

Therefore the missing number is 0.04.

Ordering Decimals

Remember: When working with decimals it is very important that you know and understand the place value of numbers.

Thousands (1000)	Hundreds (100)	Tens (10)	Ones (1)	●	Tenths ($\frac{1}{10}$)	Hundredths ($\frac{1}{100}$)	Thousandths ($\frac{1}{1000}$)
		3	**2**	**.**	**4**	**5**	**7**

For example, we can use this chart to see that the number 32.457 is made up of:

3 tens, 2 ones, 4 tenths (or 4/10), 5 hundredths (or 5/100) and 7 thousandths (or 7/1000).

We can use this understanding of place value to help us order decimal numbers.

Put these numbers in order from smallest to largest:

5.51, *3.75,* *7.35,* *5.73,* *3.77*

To answer this question, let's start by looking at the ones. There are two numbers which have the smallest number of ones: *3.75* and *3.77*

Let's first look at 3.75:

ones		$\frac{1}{10}$	$\frac{1}{100}$		ones		$\frac{1}{10}$	$\frac{1}{100}$
3	.	7	5	Look at the whole number first- the smallest whole number is **3**.	3	.	7	5
3	.	7	7	They both have **0.7** tenths	3	.	7	7
7	.	3	5	We then look at the hundredths column, one has **0.05** and the other **0.07**- therefore 3.75 is smaller.	5	.	5	1
5	.	7	3	The nest smallest large number is 7. We then look at the tenths column. One has 0.7 and one has 0.5- therefore 5.51 is smaller.	5	.	7	3
5	.	5	1	The final number is 7.35	7	.	3	5

The decimal numbers in order from smallest to largest will be: **3.75, 3.77, 5.51, 5.73, 7.35**

Let’s now try to order decimal numbers which include **thousandths** digits.

*Put the numbers below in order from **largest** to **smallest**:*

7.600	**7.675**	**7.670**	**7.556**	**6.776**

Add in 0s so that each number has the same digits

7	.	6	0	0
7	.	6	7	5
7	.	6	7	0
7	.	5	5	6
6	.	7	0	6

To order these decimals, let’s first put them all in the correct column in the place value table:

Writing them in the correct columns will help us compare the size of the numbers.

Starting with comparing the ones, it is clear that 6.776 will be the smallest number as that is the only number which hasn’t got 7 ones. We know therefore that 6.776 will come last on our list. (Remember, the question this time asks us to order from **largest** to **smallest**. Sometimes, it is easier to start with looking for the smallest numbers and then writing the numbers in reverse.)

To find the largest of the remaining numbers, let’s first look down the **tenths** column. The top three numbers have **6** in the **tenths column** but **7.556** only has **5** in the **tenths column**. Therefore, 7.556 is smaller and will be the next smallest number on our list.

We now need to compare the remaining numbers: 7.600, 7.675 and 7.670

We have already seen that the three numbers have **7 ones** and **6 tenths**. We therefore now need to look at the **hundredths** column. **7.675** and **7.67** both have **7** in the **hundredths column** but **7.6** has **nothing** in the **hundredths** column. In other words, **7.6** has **0 hundredths** and is **smaller** than 7.675 and 7.67.

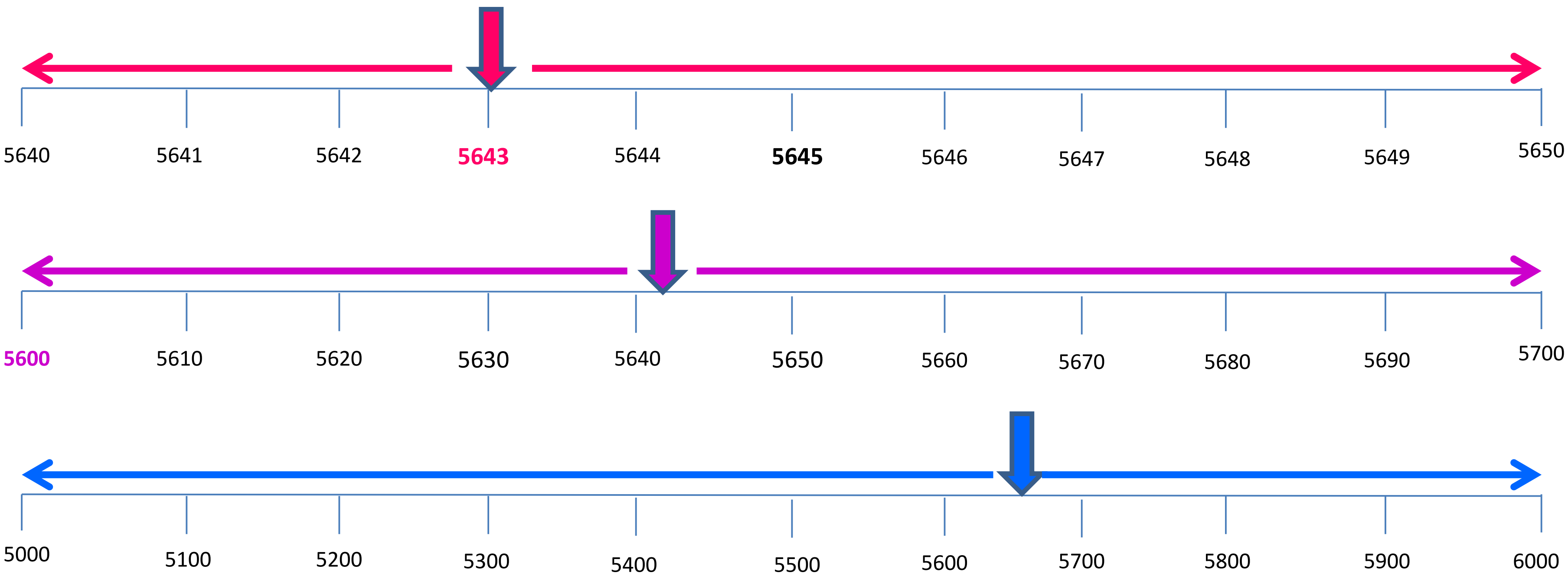
Lastly, if we compare 7.675 and 7.67, we can see that **7.675** is **larger** because **7.675** has **5 thousandths** but **7.67** has **0 thousandths**.

Another method is to write each of the numbers out with the same number of decimal places to show that there is nothing in these place value columns: **7.675, 7.670, 7.600, 7.556, 6.776**

M1c: Can round any whole number to the nearest 10, 100 and 1000

Round to the nearest 10	Round to the nearest 100	Round to the nearest 1000
What is 5,643 rounded to the nearest 10? When rounding to the nearest 10, we need to look at the digit in the ones column. 5643 3 is less than 5 so we round **down**. **5,643 rounded to the nearest 10 is 5,640.**	What is 5,643 rounded to the nearest 100? When rounding to the nearest 100, we need to look at the digits in the tens column. 5643 **40** is less than **50** so we round down. **5,643 rounded to the nearest 100 is 5,600.**	**What is 5,643 rounded to the nearest 1,000?** When rounding to the nearest 1,000, we look at the digits in the hundreds column. 5643 **600** is more than **500** so we round up. **5,643 rounded to the nearest 1000 is 6000.**
Note: Multiples of 10 end in at least **one zero**. This is a good way of checking quickly that you have rounded to the correct multiple.	**Note:** Multiples of 100 end in at least **two zeros**. This is a good way of checking quickly that you have rounded to the correct multiple.	**Note:** Multiples of 1000 end in at least **three zeros.** This is a good way of checking quickly that you have rounded to the correct multiple

Using number lines can help to visualise where the number is and how close it is to the next 10,100 or 1000



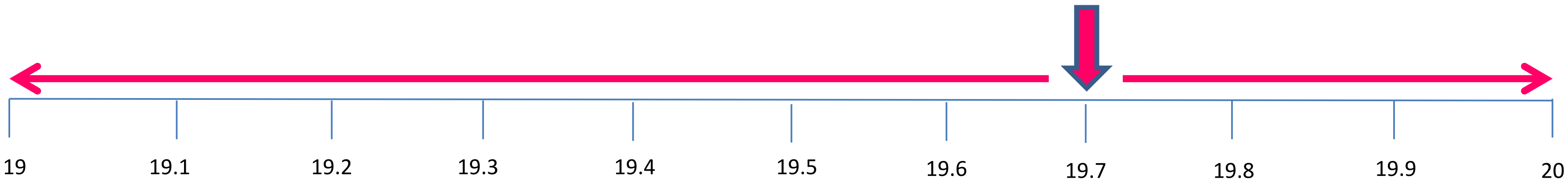
What is 7,500 rounded to the nearest 1,000?

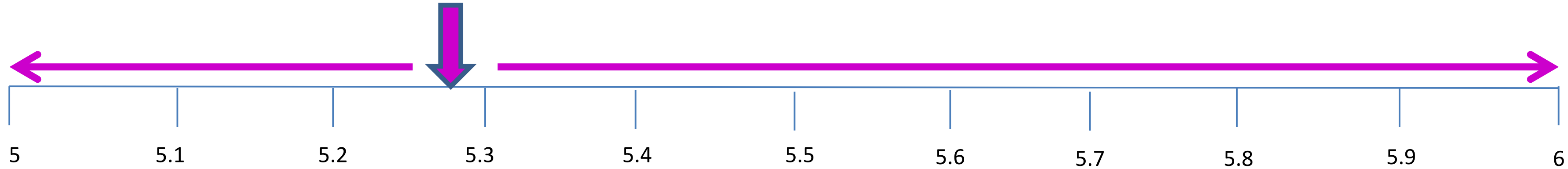
Note: When a number lies exactly half way between, we round up.

In this example 7,500 is exactly half way between 7,000 and 8,000 so we round up. 7,500 rounded to the nearest 1,000 is 8,000.

M1d: Can round decimals to the nearest whole number and to one or two decimal places

Round to the nearest whole number	Round to the nearest $^1/_{10}$	Round to the nearest $^1/_{100}$
When rounding to the nearest whole number, we look at the tenths column. **What is 19.7 rounded to the nearest whole number?** **7 tenths** is more than **5 tenths** so we round up. **19.7 rounded to the nearest whole number is 20**	**When rounding to the nearest tenth, we look at the digit in the hundredths column.** ***What is 5.28 rounded to the nearest tenth?*** **8 hundredths** is more than **5 hundredths** so we round up. **5.28 rounded to the nearest tenth is 5.3**	When rounding to the **nearest hundredth**, we look at the digit in the **thousandths column**. ***What is 6.392 rounded to the nearest hundredth?*** **2 thousandths** is less than **5 thousandths** so we round down. **6.392 rounded to the nearest hundredth is 6.39**
What is 25.68 rounded to the nearest whole number? **6 tenths** is more than **5 tenths** so we round up. **25.68 rounded to the nearest whole number is 26**	The question may be phrased differently. For example: ***What is 3.82 rounded to one decimal place?*** We still need to round to the **nearest tenth**. We, therefore, look at the digit in the **hundredths column.** **2 tenths** is less than 5 tenths so we round down. **3.82 rounded to one decimal place is 3.8**	The question may be phrased differently. For example: ***What is 12.345 rounded to two decimal places?*** We still need to round to the **nearest hundredth**. We, therefore, look at the digit in the **thousandths column.** **5 thousandths** is exactly halfway so we round up. **12.345 rounded to two decimal places is 12.34**
What is 148.39 rounded to the nearest whole number? **3 tenths** is less than **5 tenths** so we round down. **148.39 rounded to the nearest whole number is 148**		





M1e: Can use place value to multiply whole numbers by 10, 100 or 1000

When you multiply by 10 the original number gets 10 times bigger.

Hundreds	Tens	Ones
		9
	9	0

9 x 10 = 90
The 9 moves one place value column to the left

Thousands	Hundreds	Tens	Ones
	7	2	5
7	2	5	0

725 x 10 = 7250
All digits move one place value column to the left

When we multiply by 10, each digit moves one place to the left: The Ones digit moves to the Tens column, Tens moves to the Hundreds column etc. The space in the column is filled with a 0, which is called a place holder.

Multiplying by 100

When we multiply by **1**00, each digit moves **two places** to the left and the spaces in the columns are filled with a **0**, the **place holder.**

25 x 100 = 2,500

Thousands	Hundreds	Tens	Ones
		2	5
2	5	0	0

5603 x 100 = 560,300

HTH	TTH	Th	H	T	Ø
		5	6	0	3
5	6	0	3	0	0

Multiplying by 1000

When we multiply by **1,**000, each digit moves **three places** to the left and the spaces in the columns are filled with a **0**, the **place holder**

75 x 1000 =75,000

HTh	TTh	Th	H	T	Ø
	←	←	←	7	5
	7	5	0	0	0

456 x 1000 = 456,000

M	HTh	TTh	Th	H	T	Ø
	←	←	←	5	6	7
	5	6	7	0	0	0

M1g: Can use place value to divide whole numbers by 10, 100 or 1000

When you divide by 10 the original number gets 10 times smaller.

Hundreds	Tens	Ones
	9	0
		9

90 ÷ 10 = 9
The 9 moves one place value column to the right

H	T	Ø		1/10
	7	2	.	
		7		2

72 ÷ 10 = 7.2
All digits move one place value column to the right

When we divide by 10, we are making the number smaller so each digit moves 1 place to the right. Thousands move to the hundreds column, hundreds move to the tens column, tens move to the Ones column, Ones move to the tenths column. THE DECIMAL POINT DOES NOT MOVE.

Dividing by 100

When we divide by **1**<u>00</u>, each digit moves <u>**two places**</u> to the **RIGHT**

65 ÷ 100 =

H	T	Ø		1/10	1/100
	6	5			
		0	.	6	5

5603 ÷ 100 = 560,300

TTh	Th	H	T	Ø		1/10	1/100
	5	6	0	3	.		
			5	6	.	0	3

Dividing by 1000

When we divide by **1,**<u>000</u>, each digit moves <u>**three places**</u> to the **RIGHT**

75 ÷ 1000 =75,000

TTh	Th	H	T	Ø		1/10	1/100	1/100
			7	5	.			
				0	.	0	7	5

4576 ÷ 1000 = 456,000

TTh	Th	H	T	Ø		1/10	1/100	1/100
	4	5	7	6	.			
				4	.	5	7	6

M1h: Can use place value to divide decimal numbers by 10, 100 or 1000

Thousands	Hundreds	Tens	Ones	.	Tenths	Hundredths	Thousandths
2	4	0	0	.			
	2	4	0	.			
		2	4	.			
			2	.	4		

Note: Move to the right to reduce the value.

The same rules apply to decimal numbers.

- ✓ When we divide by **10**, we are making the number **smaller** so each digit moves **1** place to the **right.**
- ✓ Thousands become Hundreds, Hundreds become Tens, Tens become Ones.
- ✓ When we are dividing by **100**, we are making the number 100 times **smaller** so each digit moves **2** places to the right.
- ✓ When we are dividing by **1000**, we are making the number 1000 times **smaller** so each digit moves **3** places to the right.

	T	Ø	.	1/10	1/100	1/1000	1/10000
There will be occasions where you will need to add a **0** as a **place holder** for the answer to the calculation.	2	7	.	4			
When we divide a decimal number by 10, each digit moves one place to the right so 27.4 ÷ 10 = 2.74		2	.	7	4		
When we divide a decimal number by 100, each digit moves two places to the right so 27.4 ÷ 100 = 0.274		0	.	2	7	4	
When we divide a decimal number by 1000, each digit moves three places to the left so 27.4 ÷ 1000 = 0.0274		0	.	0	2	7	4

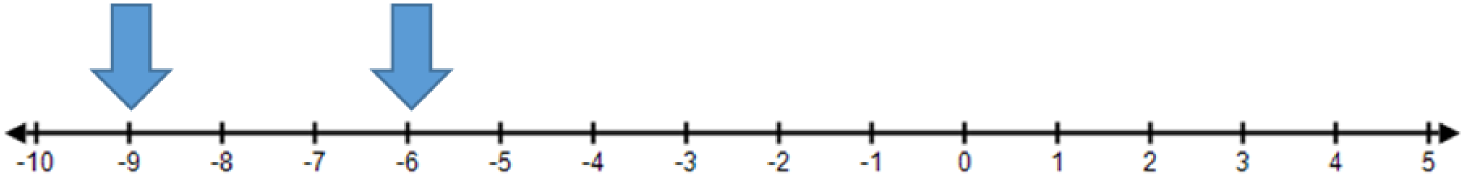
Note: Dividing by 100 is the same as dividing by 10 and 10 again. Dividing by 1000 is the same as dividing by 10, 10 and 10 again or 100 and 10.

M1i: Can use negative numbers in context and calculate intervals across zero

Remember: Negative numbers are numbers smaller than zero. Positive numbers are numbers which are greater than zero.

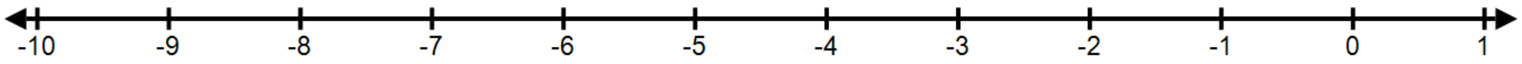
This number line shows the numbers from -10 to 5. Remember on a horizontal line, that as we move to the left, the numbers get smaller.

We can see on the number line that -9 is smaller than -6.

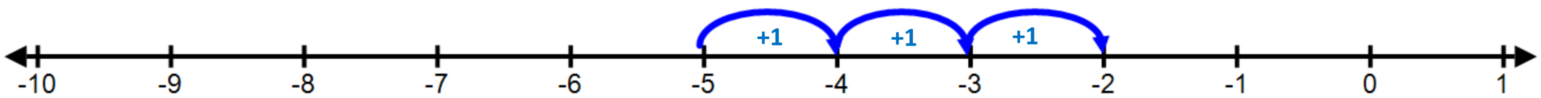


We can use number lines to help us add and subtract with negative numbers. Let's look at some examples:

Using the number line below, work out the difference between -5 and -2



To find the difference between -5 and -2, we need to count how many steps it is from -5 to -2 (**shown by the blue arrows**).

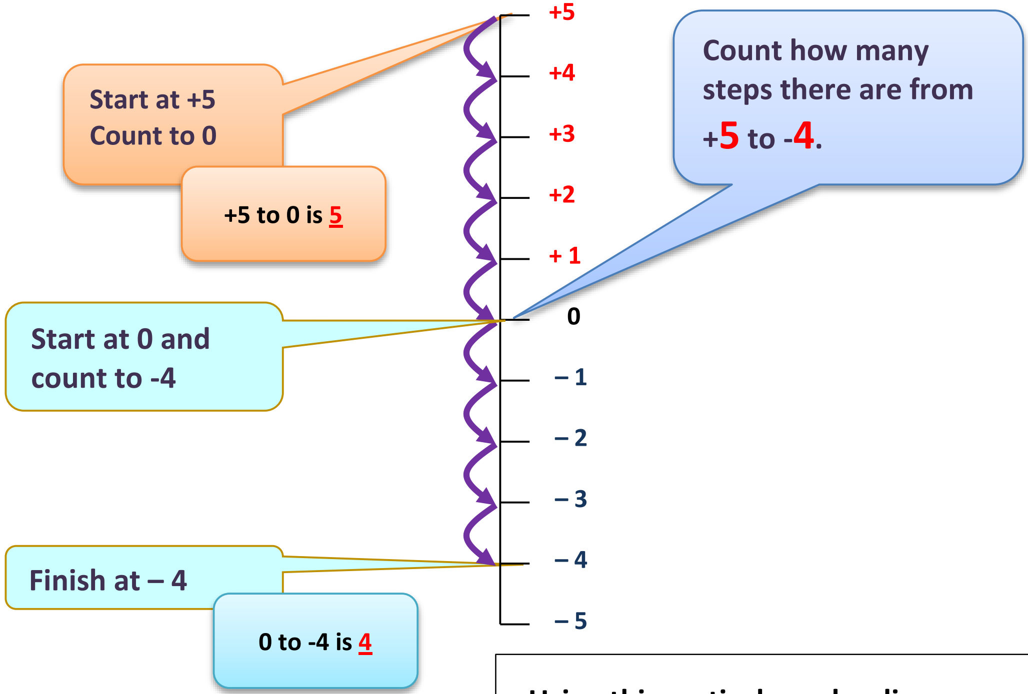


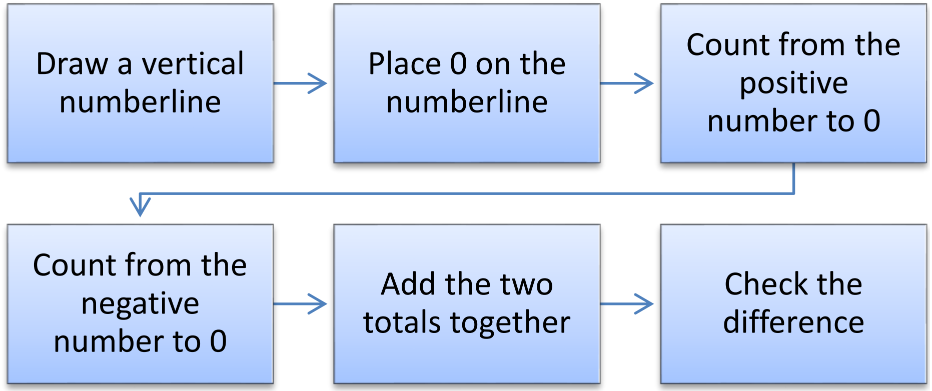
We can see that we have gone up in 3 steps. Therefore:

The difference between -5 and -2 is 3.

It is important to remember that number lines are not always horizontal. You will sometimes see vertical number lines, especially when we are talking about temperatures. Think of them looking like thermometres. Let's look at some examples

During the day the temperature reached 5˚C but at night the temperature dropped to – 4˚C. By how much did the temperature drop?



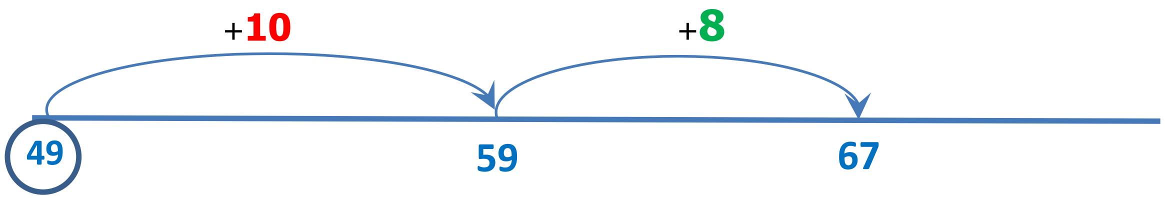
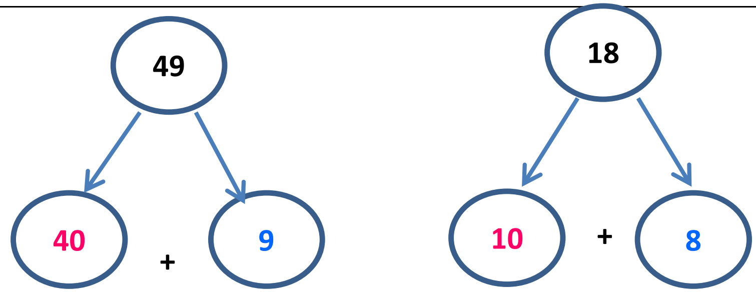
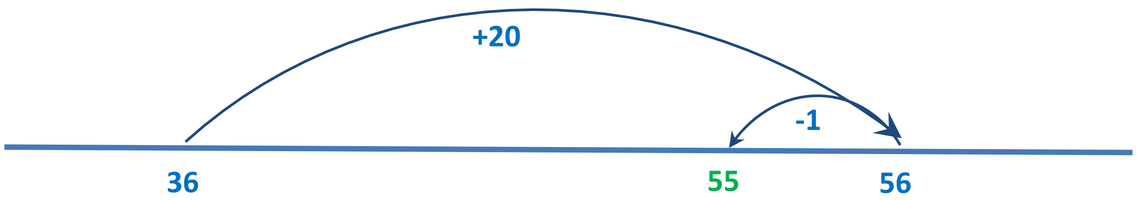


Using this vertical number line, we can see that if the temperature dropped from 5˚C to -4˚C, it dropped by 9˚C.

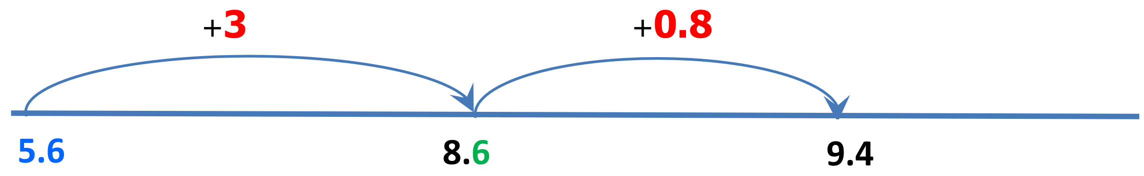
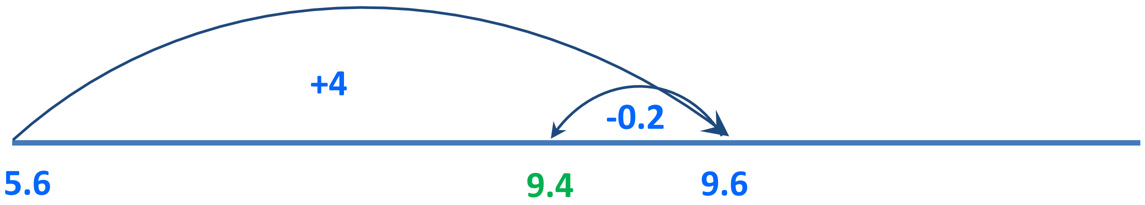
Number: Addition and Subtraction

M2a: Can use mental methods of computation for addition

Remember: when faced with any calculation we should look at the numbers involved and ask ourselves, 'can I do the calculation mentally or in my head'?	**Remember:** solving a calculation 'mentally' or 'in my head' does not mean that we cannot jot things down to help us such as the numbers involved or a number line.

Number line	**1) 49 + 18 (18 = 10 + 8)**  	✓ Draw an empty number line and write the largest number on it. ✓ Add the multiples of 10 from the second number (18) ✓ Add on the ones from the second number
Partitioning	**2) 49 + 18** **40 + 10 = 50** **9 + 8 = 17** **50 +17 = 67**  	✓ Partition the numbers into their place value groups i.e. Tens and Ones ✓ **Add the multiples of 10 together** ✓ **Add the ones together** ✓ **Total the two sums to calculate the final answer**
Rounding and adjusting	**3) 36 + 19 =**  	Look at the numbers: 19 is very close to 20- adding multiples of 10 is easier than adding 19. **20 = 19 + 1 therefore** **(36 + 20) – 1 = the answer** **56 – 1 = 55**

Adding decimal numbers

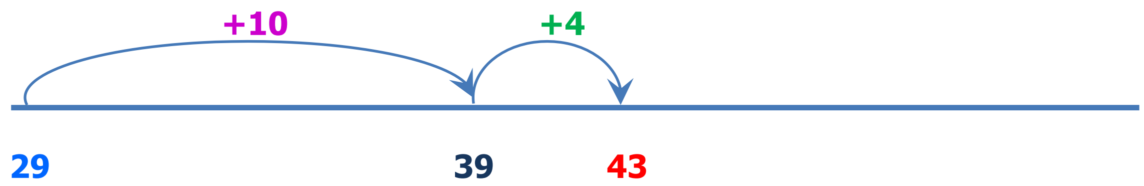
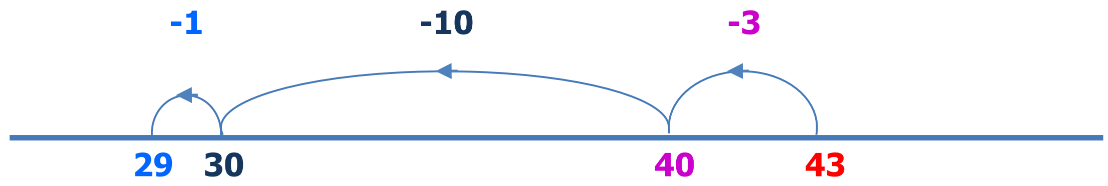
Number line	**1) 3.8 + 5.6 =** 	✓ Draw the number line ✓ Write the largest number on the line ✓ Add the whole number ✓ Add the decimal **Note: when adding the whole number the decimal stays the same**
Partitioning	**2) 3.8 + 5.6 =** **3 + 5 = 8** **0.8 + 0.6 = 1.4** **18 + 1.4 = 9.4** 	✓ Partition the numbers into their place value groups i.e. Ones and tenths ✓ Add the ones together ✓ Add the tenths together ✓ **Total the two sums to calculate the final answer**
Rounding and adjusting	**3) 3.8 + 5.6 =** 	Look at the numbers: 3.8 is very close to 4- adding whole numbers is easier than adding 3.8. **4 = 3.8 + 0.2 therefore** **(5.6 + 4) – 0.2 = the answer** **9.4 –0.21 = 9.4**

When adding more than two numbers it is important to look for numbers that make number bonds to ensure the calculation is accurate and quick

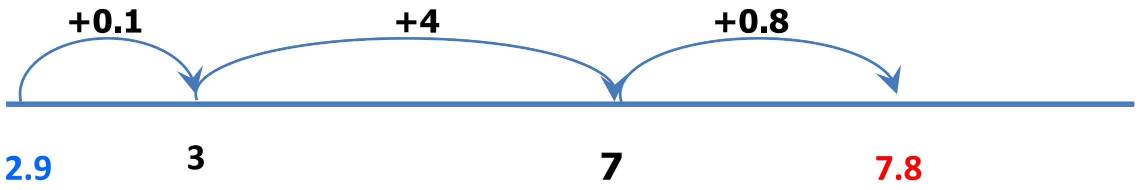
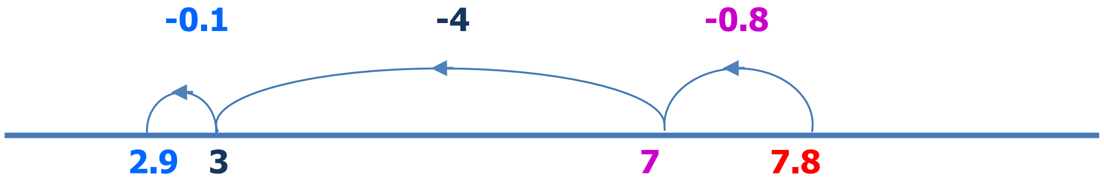
47 + 32 + 13 = → **40+(30+10)= 80** → **(7+3)+2=12** → **80+12=92**

M2b: Can use mental methods of computation for subtraction

Remember: when faced with any calculation we should look at the numbers involved and ask ourselves, 'can I do the calculation mentally or in my head'?	**Remember:** solving a calculation 'mentally' or 'in my head' does not mean that we cannot jot things down to help us such as the numbers involved or a number line.

Number line	**1) 43 - 29 = 14 (10+4)**  **Counting on (the difference between the two numbers)**	✓ **Place the smallest number on the number line** ✓ Place the largest number on the number line ✓ Count on in multiples of 10 ✓ Count on in ones ✓ Total the 'jumps' to calculate the different by counting on (10+4)
	We can also use a number line to help us do this calculation by working backwards.  	Starting with **43**, first subtract **3** which will makes **40**. Then subtract **10** which makes **30** lastly subtract **1** to get to **29**. Altogether, to get from 43 to 29 we have subtracted **3**, **10** and **1**. Therefore, 43 – 29 = 14
Partitioning	**2) 95 - 13 =** **90 - 10 = 80** **5 – 3 = 2** **So, 95 – 13 = 82** **or** **95 – 10 = 85** **85 – 3 = 82** **So, 95 – 13 = 82**	✓ Partition and Subtract multiples of 10 ✓ Subtract ones ✓ **Total the numbers** **or** ✓ Leave the largest number ✓ Subtract multiples of 10 ✓ Subtract ones gives the **answer**
Rounding and adjusting	**3) 87 – 19 =** **87 – 19 = (87 – 20) + 1** **67 + 1 = 66**	**19 + 1 = 20** We can use this to subtract 20 and then adjust by adding the 1 back on after. This makes calculating the answer easier!

Subtracting decimal numbers

Number line	**1) 7.8 – 2.9 =**  **Counting on (the difference between the two numbers)**	✓ Count on to the nearest whole number ✓ Count on in whole numbers ✓ Add on the final decimal ✓ Total the 'jumps' to calculate the answer
	We can also use a number line to help us do this calculation by working backwards.  	✓ Start with the largest number and count back ✓ Subtract the decimal to give a whole number ✓ Count back in whole numbers ✓ Count back the remaining decimal
Partitioning	**2) 9.4 – 5.8=** **9.4 – 5 = 4.4** **4.4 – 0.8 = 3.6** **So, 9.4 – 5.8 = 3.6**	✓ Subtract the whole number ✓ Subtract the decimal
Rounding and adjusting	**3) 9.4 - 5.8** We will use the rounding and adjusting strategy here. 9.4 - 6 = 3.4 (5.8 rounds to 6) We now need to adjust this because we have subtracted 0.2 too much. We therefore need to add that back on. 3.4 + 0.2 = 3.6 9.4 – 5.8 = 3.6	✓ Round one of the numbers up ✓ Subtract the whole number from the decimal ✓ Adjust

M2c: Can use efficient written methods of addition including column addition with more than 4 digits

Remember when we look at a calculation we should always ask:

- Can I do it in my head? With/without jottings?
- Do I need a written method?

Sometimes numbers are too large or there are too many numbers to calculate in our head. We need a reliable written method to help us.

Partitioning	Expanded Method	Short addition	Place value: What is the value of each digit here? For example:
728 + 546 = 728 = 700 + 20 + 8 546 = 500 + 40 + 6 1200 +60 + 14 1200 + 60 + 14 = 1274 so 728 + 546 = 1274	728 + 546 14 Add the ones 60 Add the tens 1200 Add the hundreds 1274 Total the numbers	728 + 546 1274 1	8 + 6 = 14 ----------- 14 = 1 ten (10) and 4 ones Place the ten in the 'tens' column. 40 + 20 + 10 = 70 700 + 500 is 1200

Larger numbers / **Decimal numbers**

Expanded Method (Larger numbers)	Short addition (Larger numbers)	Expanded Method (Decimal numbers)	Short addition (Decimal numbers)
45367 + 3145 + 3145 12 48512 100 400 8000 +40000 48512	45367 45367 + 3145 + 3145 48512 1 1	58.39 + 9.85 + 9.85 0.14 68.24 1.10 17.00 50.00 68.24	58.39 + 9.85 68.24 1 1 1

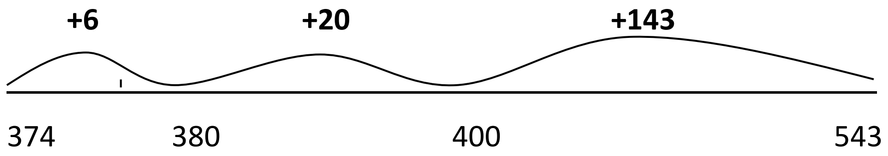
M2d: Can use efficient written methods of subtraction including column subtraction

The counting on Method

Subtraction can sometimes be calculated more easily by ‘finding the difference’ between the numbers. This can be illustrated on a number line.

543 – 374 = ?

To ‘find the difference’ between these two numbers we can count up from 374 to 543 on a number line.



Now add the steps together:

143 + 20 + 6 = 169 so **543 – 374 = 169**

Let’s do the same calculation more efficiently:

```
  543
- 374
   + 6   (380)
  + 20   (400)
 + 143   (543)
   169
```

374 + 6 = 380

380 + 20 = 400

400 + 143 = 543

so 543 – 374 = **169**

The expanded Method

Use base 10 apparatus to help with this strategy

```
 758       700   50   8
-217       200   10   7
           500 + 40 + 1 = 541
```

8 subtract 7 = 1

50 subtract 10 = 40

700 hundred subtract 200 = 500

500 + 40 + 1= the difference between 758 and 217

```
 543       400    130   13
-374       ~~500~~  ~~40~~
           300    70    4
           100+   60+   9=169
```

So, 543 – 374 = 169

3 subtract 4 gives a negative number so we need to take a 10 from the tens column.

30 subtract 70 gives a negative number so we need to take a 100 from the Hundreds column.

Expanded Method	Short subtraction	Expanded Method	Short Subtraction
700 50 8 - 200 10 7 500 + 40 + 1 = 541	758 -217 541	6523 - 654 5000 1400 110 1 6000 500 20 3 600 50 4 5000 800 60 9	5 6 14 5 11 2 1 3 - 6 5 4 5 8 6 9

Short subtraction- further examples			
5 8 2 3 1 5 6 - 4 4 0 9 2 1 4 2 6 4	1 7 8 . 1 6 6 - 3 . 7 3 1 4 . 9 3	7 5 7 8 1 3 9 9 -2 5 5 7 3 3 5 0 2 6 6 6	4 5 1 6 7 .3 5 - 9 3 .2 5 4 7 4. 10

M2e: Can add with decimals to two places (including money)

Expanded Method

T	Ø	.	1/10	1/100
	7	.	3	6
	6	.	4	2
	0	-	0	8
	0	.	7	0
1	3	.	0	0
1	3	.	7	8

Short addition

T	Ø	.	1/10	1/100
	7	.	3	6
	6	.	4	2
1	3	.	7	8

Expanded Method

T	Ø	.	1/10	1/100
3	6	.	7	4
5	2	.	6	5
	0	-	0	9
	1	.	3	0
	8	.	0	0
8	0	.	0	0
8	9	.	3	9

Short addition

T	Ø	.	1/10	1/100
3	6	.	7	4
5	2	.	6	5
8	9	.	3	9
	1			

Adding decimals (including money) is just the same as whole numbers. Just make sure that you line up the decimal points and fill in the numbers in the correct place value columns. Put **0s** as place value holders to stop you getting confused…easy as that!

M2f: Can subtract with decimals to two places (including money)

8.62 – 4.83 =

(Counting on method)

8.62
- 4.83
+ 0.07 (4.9) — Counting on to the nearest tenth
+ 0.10 (5) — Counting on to the nearest unit
+3.62 (8.62)
3.79

(Expanded)

^{7}~~8~~	$^{1.5}$~~0.6~~	0.0^{1}2
- 4	0.8	0.03
3	+ 0.7 +	0.09

(Short subtraction)

^{7}~~8~~.^{5}~~6~~^{1}2
- 4. 8 3
3. 7 9

Subtraction with money (counting on)

£27.50 - £14.24 =

2 7 . 5 0
1 4 . 2 4
0. 7 6 (15) — **To get from 14.24 to 15 we need to add 0.76**
1 2 . 5 0 (27.50) — **To get from 15 to 27.50 we need to add 12.50**
1 3 . 2 6 — **In total, to get from 14.24 to 27.50, we have added 13.26**

So, £27.50 + £14.24 = £13.26

Short Subtraction with money

T	Ø	.	1/10	1/100
2	7	.	^{4}~~5~~	^{1}0
1	4	.	2	4
1	3	.	2	6

With money, line up the decimal points and place the numbers into the correct columns.
The methods for subtraction work for all numbers (decimals including money!)

Number: Multiplication and Division

M3b: Can use tables and place value with multiples of 10

We can use multiplication facts to help us understand other times tables questions:

E.g. if we know that 6 x 4 = 24 we also know that 6 x 40 = 240 and that 60 x 40 = 2400

60 is ten times larger than 6. Our answer will also be ten times larger.

H	T	U	.	10th	100th
		6	.		
	6 ←	0	.	0	0

240 is ten times larger than 24, so **4 x 60 = 240**

H	T	U	.	10th	100th
	2	← 4	.	0	0
2 ←	4	0	.	0	0

2400 is ten times larger than 240 so **40 X 60 = 2400**

Th	H	T	U	.	10th	100th
		2	4	.	0	0
2 ←	4 ←	0	0	.	0	0

The same principles apply with division

6 x 8 = 48 480 ÷ 6 = 80 480 ÷ 8 = 60	If the question is how many **8s** in 480 you can use your knowledge of 6 x **8** = 48 to help you figure it out. 48 is 10 x smaller than 480 480 is 10 x larger than 48 480÷6 =80 480 ÷8 = 60	**7 x 3 = 21** **210÷7 =30** **210÷3= 70**	If the question is how many 7s in 210 you can use your knowledge of 3x7-21 to help 21 is 10x smaller than 210 210 is 10 x larger than 21

3,500 ÷ 7 =	We know that 35 ÷ 7 = 5. 3500 is 100 times larger than 35.	So the answer is: 3,500 ÷ 7 = 500
6,300 ÷ 90 =	We know that 63 ÷ 9 = 7. 6300 is 100 times larger than 63.	So the answer is: 6,300 ÷ 90 =70
490 ÷ 7 =	We know that 49 ÷ 7 = 7. 6300 is 100 times larger than 490.	So the answer is: 490 ÷ 7 =

M3c:	Can use mental methods of computation for multiplication

Doubling			Partitioning		
To double 96 we can partition and multiply by 2: 96 = 90 + 6 90 x 2 = 180 6 x 2 = 12 180 + 12 = 192 **so double 96 is 192**	**Double a 3 digit number:** 174 = 100 + 70 + 4 100 x 2 = 200 70 x 2 = 140 4 x 2 = 8 200 + 140 + 8 = 348 **so double 174 is 348**	**£8.90 = £8.00 + 90p** £8.00 x 2 = £16.00 90p x 2 = £1.80 £16.00 + £1.80 = £17.80 **so double £8.90 is £17.80**	**Multiply 35 by 6.** 35 = 30 + 5 30 x 6 = 180 5 x 6 = 30 180 + 30 = 210 **so 35 x 6 is 210**	**What is 134 x 6?** 134 = 100 + 30 + 4 100 x 6 = 600 30 x 6 = 180 4 x 6 = 24 600 + 180 + 24 = 804 **so 134 x 6 is 804**	Multiply 4.8 by 7 4.8 = 4 + 0.8 4 x 7 = 28 0.8 x 7 = 5.6 28 + 5.6 = 33.6 **so 4.8 x 7 is 33.6**

Using easier known facts			
x4- double and double again	x5 we can x10 and then ½ (÷2)	x50 times by 100 then ½	x25 times by 100 then ¼ (÷4)
24 x 4 24 = 20 + 4 20 x 2 = 40 4 x 2 = 8 **so 24 x 4 is 96**	***32 x 5*** 32 x 10 = 320 320 ÷ 2 = 160 **so 32 x 5 = 160**	***32 x 50*** 32 x 100 = 3200 3200 ÷ 2 = 1600 **so 32 x 50 = 1600**	***25 x 18*** 18 x 100 = 1800 1800 ÷ 4 = 450 **so 25 x 18 = 450**

M3d:	Can use mental methods of computation for division

Halving a 2 digit number	Halving a 3 digit number	Halve a decimal
78 We know that halving is the same as dividing by 2. To halve 78 we can partition and divide by 2: **78 = 70 + 8** 70 ÷ 2 = 35 8 ÷ 2 = 4 35 + 4 = 39 **So half of 78 is 39**	**158** 100 = 100 + 50 + 8 100 ÷ 2 = 50 50 ÷ 2 = 25 8 ÷ 2 = 4 550 + 25 + 4 = 79 **So half of 158 is 79**	**£7.60** £7.60 = £7.00 + 60p £7.00 ÷ 2 = £3.50 60p ÷ 2 = 30p £3.50 + 30p = £3.80 **So half of £7.60 is £3.80**

Using easier known facts			
÷4 (divide by 4)	÷ 8 (divide by 8)		
64 ÷ 4 64 = 60 + 4 60 ÷ 2 = 30 4 ÷ 2 = 2 32 = 30 + 2 30 ÷ 2 = 15 2 ÷ 2 = 1	**264 ÷ 8** Half of 264 is 132 Half of 132 is 66 Half of 66 is 33 **so 264 ÷ 8 = 33**	**What is 84 ÷ 7?** We know that 10 multiplied by 7 is 70. This fact helps us here. **84 can be partitioned into 70 and 14.** 84 = 70 + 14 70 ÷ 7 = 10 14 ÷ 7 = 2	**What is 96 ÷ 6?** 96 = 60 + **36** 60 ÷ 6 = 10 **36** ÷ 6 = 6 10 + 6 = 16 **So 96 ÷ 6 = 16**

15 + 1 = 16 **So 64 ÷ 4 = 16**		10 + 2 = 12 **So 84 ÷ 7 = 12**	

M3e:	**Can use efficient written methods of multiplication including short and long multiplication**

Multiplication is easier when you a) know your times tables facts and b) can set your work out correctly!

Expanded Multiplication

Th	H	T	Ø	
	2	6	8	
X			4	
		3	2	= 4 x 8
	2	4	0	= 4 x 60
	8	0	0	= 4 x 200
1	0	7	2	

SET your calculation our correctly to avoid errors

DON'T forget the value of the digits when multiplying. 268=200+60+8

You MUST make sure that you multiple each digit in the top row (268) by the number in the bottom row (4)

Short multiplication

Th	H	T	Ø
	2	6	8
X			4
1	0	7	2
	2	3	

4 x 8 = 32
Put 3 in the Ts column
Put the 2 in the Øs column

→

4 x 60 = 240
Put the 2 in the H column
The 4 in the Ts column
The 0 in the Ø column

→

2 x 200= 800
800+200 =100

Expanded Multiplication

HTh	TTh	Th	H	T	Ø
			5	6	3
X				2	7
				2	1
			4	2	0
		3	5	0	0
				6	0
		1	2	0	0
	1	0	0	0	0
	1	5	2	0	1
		1	1		

Expanded Multiplication

HTh	TTh	Th	H	T	Ø
		6	1	3	4
X				5	2
					8
				6	0
			2	0	0
	1	2	0	0	0
			2	0	0
		1	5	0	0
		5	0	0	0
3	0	0	0	0	0
3	1	8	9	6	8

Expanded Multiplication

HTh	TTh	Th	H	T	Ø
		3	6	0	4
x				4	6
				2	4
					0
		3	6	0	0
	1	8	0	0	0
			1	6	0
				0	0
	2	4	0	0	0
1	2	0	0	0	0
1	6	5	7	8	4
	1				

Short Multiplication

HTh	TTh	Th	H	T	Ø
			5	6	3
X				2	7
		3	9	4	1
			4	2	
	1	1	2	6	0
		1	1	6	
	1	5	2	0	1
		1	1		

Short Multiplication

HTh	TTh	Th	H	T	Ø
		6	1	3	4
X				5	2
	1	2	2	6	8
3	0	6	7	0	0
		1	2		
3	1	8	9	6	8

Short Multiplication

HTh	TTh	Th	H	T	Ø
		3	6	0	4
x				4	6
	2	1	6	2	4
		3		2	
1	4	4	1	6	0
	2		1		
1	6	5	7	8	4

M3f:	**Can use efficient written of division including short and long division**

When we are dividing large numbers by a single digit we can use **<u>short division</u>**.

84 ÷ 6 =**14** **1 4** 6 ⟌ 8 ^{2}4	1) We say, 'how many 6s in 8?' The answer is 1 so we put the 1 above the 8 because there is only 1 group of 6 in 8 with a remainder of 2. 2) 1 x 6 = 6 so we have a remainder of 2. We put this in front of the 4 to make 24. 3) We say, 'how many 6s in 24?' The answer is 4. We put this above the 24 next to the 1. 4) 14 is our answer. 5) So, 84 ÷ 6 = **14**
260 ÷4 = **65** **0 6 5** 4 ⟌ 2 ^{2}6 ^{2}0	1) We say, 'how many 4s in 0?' The answer is 0 so we put the 8 above the 8 2) We move the 2 next to the 6 to make 26. 3) We say, 'how many 4s in 26?' The answer is 6. We put this above the 26 next to the 0. There is a remainder of 2 4) We put the 2 next to the 0 to make 20 5) We say, 'how many 5s in 20?' The answer is 5. We put the 5 above the 2- next to the 6 6) 65 is our answer. 7) So, 260 ÷ 4 = **65**
954 ÷ 7=**136 r2** **1 3 6 r2** 7 ⟌ 9 ^{2}5 ^{4}4	1) How many 7s in 9? The answer is 1 remainder 2. We put 1 above the 9 and a 2 in front of the 5. 2) How many 7s in 25? The answer is 3 remainder 4. We put 3 above the 5 and a 4 in front of the 4. 3) How many 7s in 44? The answer is 6 remainder 2. We put the 6 above the 4 and the remainder next to it. 4) Our answer is 136 r 2. 5) So 954 ÷ 7 = **136 r 2**.

Remainders as fractions:

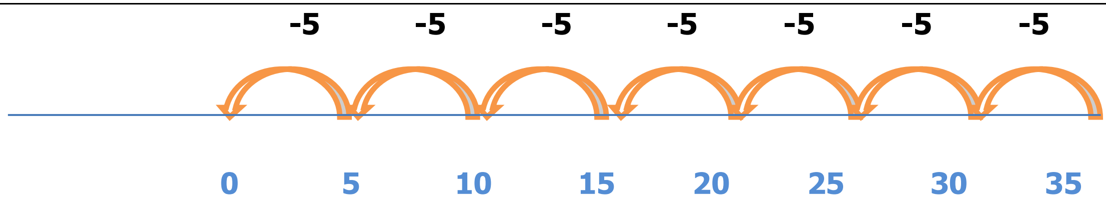
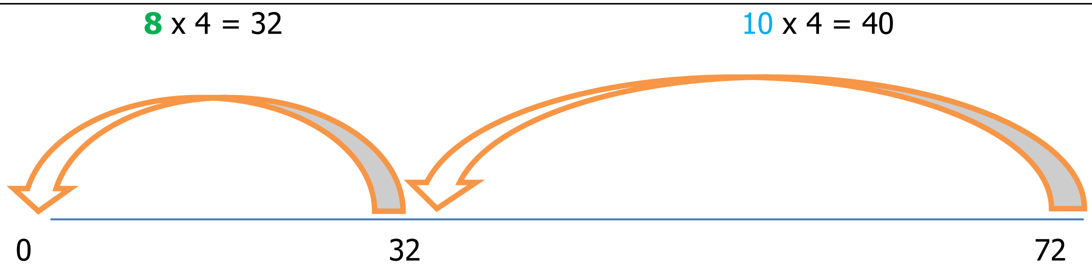
954 ÷ 7 = **136 r 2**.
We can write the remainder as a fraction. In this example it would be:

136 and $\frac{2}{7}$ because 7 is our divisor and we only have 2 out of a possible group of 7.

So 954 ÷ 7 = **136** $\frac{2}{7}$

Chunking: The Chunking Method as a step towards understanding long division

NOTE: To understand the chunking method, we must understand division as grouping as well as sharing. We can illustrate this using a simple division fact that we already know. Let's look at an example

35 ÷ 5 =		We have subtracted 7 groups of 5 **so 35 ÷ 5 = 7** Note: We can physically use 35 cubes, alongside the number line, to illustrate the process of subtracting groups of 5 from 35. To help us divide larger numbers, we can '**chunk'** or '**group'** the divisor
72÷4=	 **We have subtracted 10 groups and then 8 groups of the divisor. This totals 18.** **Therefore, 72 ÷ 4 = 18**	**Using the chunking method:** ***72 ÷ 4= 18*** 4 | **72** -40 (**10** x 4) **32** -32 (**8** x 4) **0**

We need to use our multiplication and division facts and be able to multiply by 10 and 100. We also need to subtract. We have to use a lot of skills to divide efficiently!

Chunking

468 ÷ 3=

```
3 | 468
    -300   (100x3)
    ----
     168
    -150   (50x3)
    ----
      18
    - 18   (6x3)
    ----
       0
```

100+50+6=156

Therefore, 468 ÷ 3 = 156

You want to subtract the largest amount you can each time, using number facts you can easily work out.

It is easiest to work with multiples of 10 and 100.

NOTE: keep the divisor lined up and the multiples lined up

This will help with efficient calculations at the end

768 ÷ 32=

```
32 | 768
     -640 (20x32)
     ----
      128
     -128  (4x32)
     ----
        0
```

20 + 4 = 24

Therefore, 768 ÷ 32 = 24

Division is complicated. It is always a good idea to estimate first.

Here we could partition 768 into 700 and 68.

There are roughly 3 groups of 32 in 100, so we would have 7 lots of 3 in 700. This is 21 plus an extra 2 groups from the 68.

We should expect an answer of roughly 23.

Long division

653 ÷ 24 =

```
        2
2 4 | 6 5 3
      4 8
      ---
      1 7
```

1. We say 24 divided into 6 which is not an easy division so we say 24 divided into 65.
2. 65 divided by 24. The answer is 2 with a remainder. To calculate the remainder we need to work out what is left if you subtract 2 groups of 24 from 65.
3. This time we put the 2 above the 5 and write two groups of 24 beneath the 65 so 48 is written beneath 65. We then subtract 48 from 65. The remainder of 17 is written below the 48.

```
        2
2 4 | 6 5 3
      4 8  ↓
      ---
      1 7 3
```

We also bring down the 3 from the calculation above.

We then divide 173 by 24 which is 7 with a remainder.

To calculate the remainder, we subtract 7 groups of 24 from 173.

```
        2 7 r5
2 4 | 6 5 3
      4 8  ↓
      ---
```

24 x 10 = 240
24 x 5 = 120
24 x 6 = 144
24 x 7 = 168

So 653 ÷ 24 = 27 r 5

We put the 7 at the top next to the 2 and write the 7 groups of 24 beneath the 173. We then subtract 168 from 173. The remainder of 5 is written next to the answer of 27	1 7 3 1 6 8 ——— 5	**Chunking or long division????**

M3g: Can multiply a simple decimal by a single digit

Grid method is a good way to first begin multiplying decimals by a whole number

32.6 x 5

X	30	2	0.6
5	150	10	3

5x 30=150 5x2=10 5x 0.6= 3

(0.6+0.6+0.6+0.6+0.6=3)

5 lots of 0.6

150 + 10 + 3 = 163

32.6 x 5 = 163

Th	H	T	Ø	.	$^1/_{10}$	
		3	2	.	6	
x			5	.	0	
			3	.		= 5 x 0.6
		1	0	.		= 5 x 2
	1	5	0	.		= 5 x 30
	1	6	3	.		

34.92 x 3 =

H	T	Ø	.	$^1/_{10}$	$^1/_{100}$	
	3	4	.	9	2	
x		3	.	0	0	
		0	.	0	6	= 3 x 0.02
		2	.	7		= 3 x 0.9
	1	2	.			= 3 x 4
	9	0				=3 x 90
1	0	4	.	7	6	

Leading to…

Leading to…

32.6 x 5 = 163

Th	H	T	Ø	.	$^1/_{10}$
		3	2	.	6
x			5	.	0
	1	6	3	.	
		1	3		

34.92 x 3 =

H	T	Ø	.	$^1/_{10}$	$^1/_{100}$
	3	4	.	9	2
x		3	.	0	0
1	0	4	.	7	6
	1	2			

M3i: Can identify factors and common factors

Let us see if we can find two numbers which multiply together to make 6:

We know that 2 multiplied by 3 makes 6. Therefore, we say that 2 and 3 are **FACTORS** of 6.
But 1 x 6 = 6 as well so 1 and 6 are also **FACTORS** of 6
It is best if we work systematically to find factor **pairs**.

Factors of 24

1 and 24
2 and 12
3 and 8
4 and 6

Factors of 18

1 and 18
2 and 9
3 and 6

Therefore the factors of 24 are: **1**,**2,** **3**, 4, 6, 8, 12 and 24

Therefore the factors of 18 are: **1, 2, 3**, **6,** 9 and 18

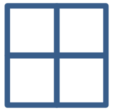
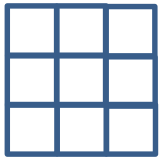
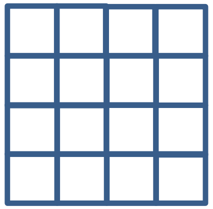
Common Factors of both 24 and **18** are: **1, 2, 3** and **6**

Lowest common factor (LCM) is 2 (we discount 1)

Highest common factor (HCF) is 6

M3j: Can recognise and describe square numbers

We say that 25 is a **SQUARE** number because both factors of **25** are the same (**5x5**).
If you multiply any number by itself, the answer you get will be a square number.
Let's look at some more examples of square numbers:

1 x 1	2 x 2	3 x 3	4 x 4
			
The first square number is 1. We know this because 1 x 1 = 1	**2 x 2 = 4 therefore, we know that 4 is a square number**	**3 x 3 = 9 therefore, we know that 9 is a square number**	**4 x 4 = 16 therefore, we know that 16 is a square number**

The first 10 square numbers are:

1	**4**	**9**	**16**	**25**	**36**	**49**	**64**	**81**	**100**	**You need to learn and recognise these numbers**
1^2	2^2	3^2	4^2	5^2	6^2	7^2	8^2	9^2	10^2	**It is also important to know the notation used when working with square numbers.**

We say that 5 squared is 25 because 5 x 5 = 25.

In mathematics we write this as 5^2 = 25.

M3k: Can recognise and identify prime numbers

Remember: A prime number is a number that has only 2 factors (1 and itself).
For example 13 is a prime number as the only factors of 13 are 1 and 13.

Prime factor: a factor of a number that also happens to be a prime number. **For example** 7 is a prime factor of 21 because 7 is a factor of 21 and 7 is a prime number. Its only factors are 1 and 7.

We should know and recognise all the **prime numbers up to 20**. These are:

2 (the only even prime number)	**3**	**5**	**7**	**11**	**13**	**17**	**19**

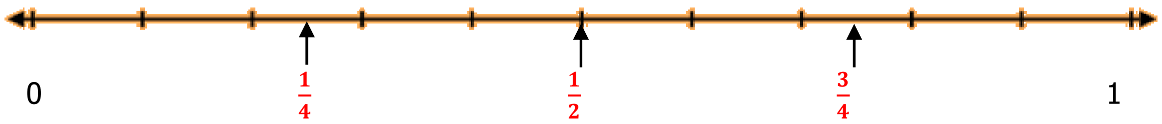
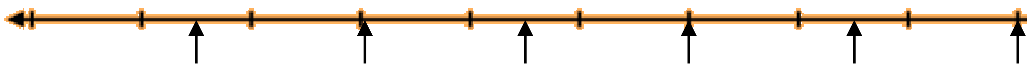
Number: Fractions, Decimals and percentages

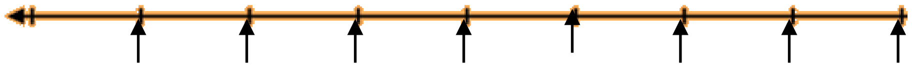
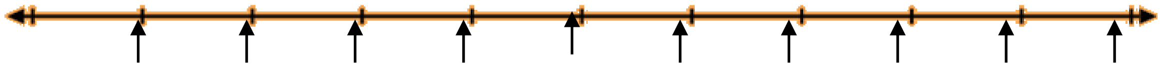
M5a: Can identify, name and write equivalent fractions of a given fraction represented visually

$\frac{3}{4}$

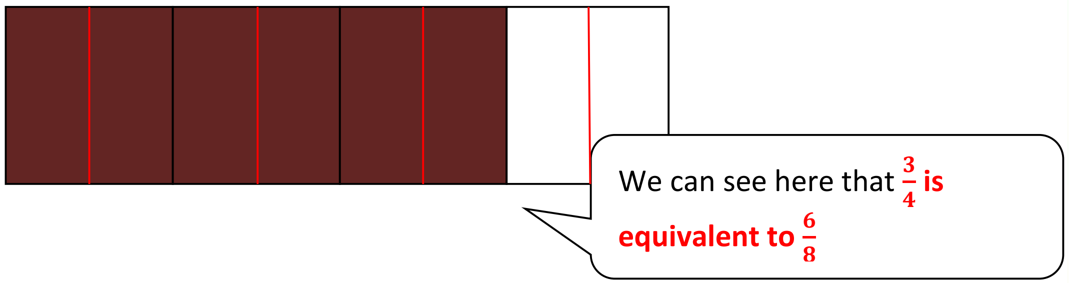
The numerator (the number at the top) tells us how many pieces we have.

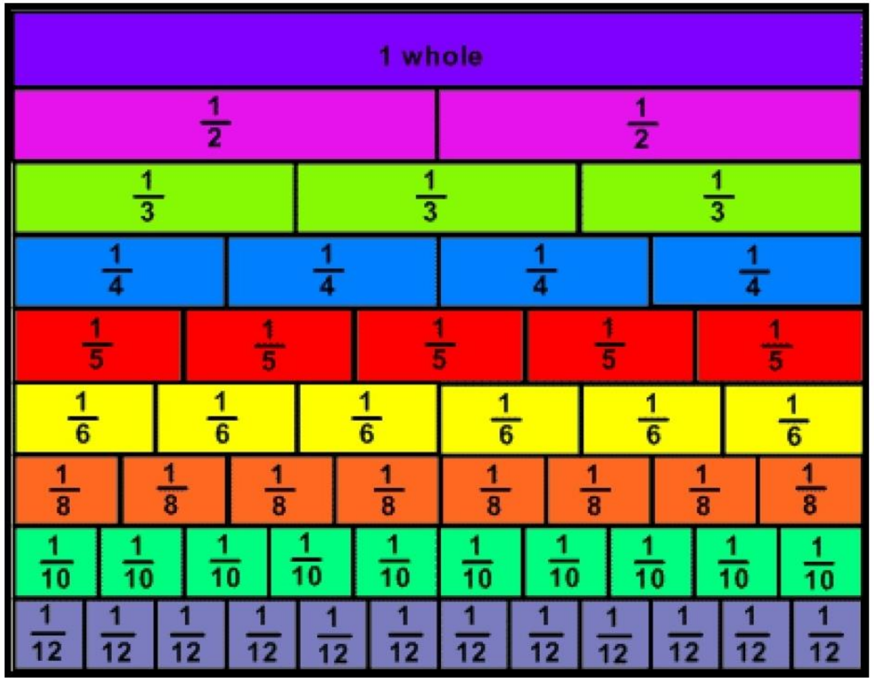
The denominator (the number at the bottom) tells us how many equal parts the whole (shape, number, quantity) has been divided into.

You should be familiar with the following fractions: 1/4 1/2 and 3/4 are shown on this number line.	
You should also be familiar with thirds and sixths	

	0 $\frac{1}{6}$ $\frac{1}{3}$ or $\frac{2}{6}$ $\frac{3}{6}$ or $\frac{1}{2}$ $\frac{2}{3}$ or $\frac{4}{6}$ $\frac{5}{6}$ $\frac{3}{3}$ or $\frac{6}{6}$ or 1
You should also be familiar with eighths (and quarters)	 0 $\frac{1}{8}$ $\frac{1}{4}$ or $\frac{2}{8}$ $\frac{3}{8}$ $\frac{2}{4}$ or $\frac{4}{8}$ or $\frac{1}{2}$ $\frac{5}{8}$ $\frac{3}{4}$ or $\frac{6}{8}$ $\frac{7}{8}$ $\frac{4}{4}$ or $\frac{8}{8}$ or 1
You should also be familiar with fifths and tenths	 0 $\frac{1}{10}$ $\frac{1}{5}$ or $\frac{2}{10}$ $\frac{3}{10}$ $\frac{4}{10}$ or $\frac{2}{5}$ $\frac{5}{10}$ or $\frac{1}{2}$ $\frac{6}{10}$ or $\frac{3}{5}$ $\frac{7}{10}$ $\frac{8}{10}$ or $\frac{4}{5}$ $\frac{9}{10}$ $\frac{10}{10}$ or $\frac{5}{5}$ or 1

You will see that some fractions are **equivalent** to each other. This means that their value is the same. For example $\frac{3}{4}$ **is equivalent to** $\frac{6}{8}$ as we could see on the number line above. Let's look at these equivalent fractions represented in a different visual way. Imagine the drawing below is a chocolate bar.





$\frac{1}{4}$	$\frac{2}{8}$ $\frac{3}{12}$ $\frac{4}{16}$ $\frac{5}{20}$ *and* $\frac{10}{40}$	**These fractions are all equivalent to** $\frac{1}{4}$
$\frac{1}{5}$	$\frac{2}{10}$ $\frac{3}{15}$ $\frac{4}{20}$ $\frac{6}{30}$ *and* $\frac{10}{50}$	**These fractions are all equivalent to** $\frac{1}{5}$
$\frac{1}{8}$	$\frac{2}{16}$ $\frac{3}{24}$ $\frac{4}{32}$ $\frac{5}{40}$ *and* $\frac{10}{80}$	**These fractions are all equivalent to** $\frac{1}{8}$

Hint: knowing your multiplication and division facts will help you find equivalent fractions quickly and efficiently!

M5b:	**Can use common factors to simplify fractions**

When fractions are equivalent we know that the equivalent fractions have the same value. For example if I had $\frac{1}{2}$ of a chocolate bar or $\frac{4}{8}$ of the same chocolate bar, I would have exactly the same amount of chocolate.

What do equivalent fractions have in common? We know that fractions are closely linked to division. $\frac{1}{2}$ tells us that something has been divided into 2 equal parts and we have 1 out of 2 equal parts. $\frac{4}{8}$ tells us that something has been divided into 8 equal parts and we have 4 out of 8 equal parts (equal to ½).

Remember: A factor is a number that divides into another number with no remainders. For example 4 is a factor of 12. A common factor is a factor that two larger numbers have in common. For example 4 is a factor of 12 and 16 because 4 divides into 12 and also divides into 16.

To simplify a fraction, we must find a factor that the numerator and denominator have in common. Let's look at an example:

1. Simplify $\frac{4}{8}$

The numerator is 4 and the denominator is 8. To simplify this fraction we must find a common factor. Both 4 and 8 have the factor 4 in common. We must now divide both the numerator and the denominator by 4.

$\frac{4 \div 4}{8 \div 4} = \frac{1}{2}$	$\frac{4}{8}$ can be simplified to $\frac{1}{2}$.	The fraction is now in its simplest form. A fraction can be put in its simplest form and be simplified. To simplify a fraction in one step, it is necessary to find the largest common factor.
Both fractions are equivalent. (See fraction wall)		

2. Simplify $\frac{5}{20}$

$\frac{5 \div 5}{20 \div 5} = \frac{1}{4}$	$\frac{5}{20}$ can be simplified to $\frac{1}{4}$	1) Look for the largest common factor of 5 and 20. 2) 5 is the largest common factor of 5 and 20. 3) Divide both 5 and 20 by 5
Both fractions are equivalent.		

3. Simplify $\frac{18}{36}$

$\frac{18 \div 9}{36 \div 9} = \frac{2}{3}$	$\frac{18}{36}$ can be simplified to $\frac{2}{3}$	1) Look for the largest common factor of 18 and 36. 2) 9 is the largest common factor of 18 and 36.

Both fractions are equivalent. (See fraction wall)	3) Divide both 18 and 36 by 9.

Hint: knowing your multiplication and division facts will help you simplify fractions quickly and efficiently!

M5c: Can compare and order fractions

When fractions have a common denominator, it is straightforward to compare them and put them in order.	
Put the following fractions in order from smallest to largest: $\frac{4}{5}$ $\frac{2}{5}$ $\frac{3}{5}$	**Clearly, the correct order for these fractions is:** $\frac{2}{5}$ $\frac{3}{5}$ $\frac{4}{5}$
Sometimes the fractions do not have a common denominator but the fractions are familiar. For example we know that $\frac{1}{4}$ is smaller than $\frac{1}{2}$ and that $\frac{1}{3}$ is bigger than $\frac{1}{6}$ etc. These fractions are familiar and can be compared easily on a number line.	

Which is the smallest fraction? $\frac{3}{4}$ **or** $\frac{2}{3}$**?**

$\frac{3}{4} = \frac{?}{12}$ $\frac{2}{3} = \frac{?}{12}$

Before we can compare these fractions, we must make sure they both have a common denominator. To do this, we need to find the lowest common multiple of all the denominators. The lowest common multiple is the lowest number that both numbers go into. In this case, we need to find the lowest common multiple of 3 and 4.

What is the lowest number that these numbers go into? The answer is 12. We now need to write each fraction with 12 as a denominator.

		x 3	

$\frac{3}{4} = \frac{?}{12}$	To find an equivalent fraction, we need to look at what we have multiplied the denominator by to get 12. Whatever we have multiplied the denominator by, we need to multiply the numerator by	$\mathbf{\frac{3}{4} = \frac{?}{12}}$	$\frac{3}{4} = \frac{9}{12}$

Order the following fractions from smallest to largest:

$$\frac{5}{16} \qquad \frac{1}{8} \qquad \frac{1}{2} \qquad \frac{3}{4} \qquad \frac{1}{4}$$

Before we can compare these fractions, we must make sure they all have a common denominator. To do this, we need to find the **lowest common multiple** of all the denominators. The lowest common multiple is the lowest number that all the other numbers go into. In this case, we need to find the lowest common multiple of **16, 8, 2, and 4.** What is the lowest number that all of these numbers go into? **The answer is 16.** We now need to write each fraction with 16 as a denominator. (The first fraction already has a denominator of 16.)

$\mathbf{\frac{1}{8} = \frac{?}{16}}$	$\mathbf{\frac{1}{2} = \frac{?}{16}}$	$\mathbf{\frac{3}{4} = \frac{?}{16}}$	$\mathbf{\frac{1}{4} = \frac{?}{16}}$
When finding equivalent fractions, we multiply or divide the numerator and denominator by the same number.			
x 2 $\frac{1}{8} = \frac{?}{16}$ x 2	x 8 $\frac{1}{2} = \frac{8}{16}$ x 8	x 4 $\frac{3}{4} = \frac{12}{16}$ x 4	x 4 $\frac{1}{4} = \frac{4}{16}$ x 4
$\mathbf{\frac{1}{8} = \frac{2}{16}}$	$\mathbf{\frac{1}{2} = \frac{8}{16}}$	$\mathbf{\frac{3}{4} = \frac{12}{16}}$	$\mathbf{\frac{1}{4} = \frac{4}{16}}$

Now that they have a common denominator, we can now compare the fractions and put them in order:

$$\frac{2}{16} \quad \frac{4}{16} \quad \frac{5}{16} \quad \frac{8}{16} \quad \frac{12}{16}$$

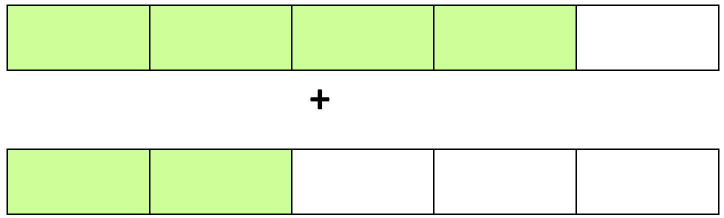
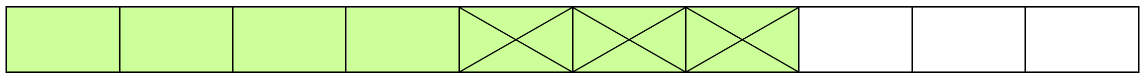
Don't forget to give the original fractions in your answer:

The fractions in order from smallest to largest are:

$$\frac{1}{8} \quad \frac{1}{4} \quad \frac{5}{16} \quad \frac{1}{2} \quad \frac{3}{4}$$

M5d: Can add and subtract fractions

When fractions have a common denominator, it is straightforward to compare them and also to add and subtract them. For example,

Addition	$\frac{4}{5} + \frac{2}{5} = \frac{6}{5}$	$\frac{6}{5}$ can also be written as $1\frac{1}{5}$	We can see this represented visually too: 
Subtraction	$\frac{7}{10} - \frac{3}{10} = \frac{4}{10}$	Simplify the fraction… $\frac{4}{10} = \frac{2}{5}$	We can see this represented visually too:  7 out of 10 pieces is equal to $^{7}/_{10}$ if we subtract 3 of those pieces we are left with $^{4}/_{10}$

Sometimes fractions don't have a common denominator. If this is the case we must first convert the fractions so that they do have a common denominator. **We CANNOT add fractions which do not have the same denominator.**

X2

$$\frac{5}{16} + \frac{1}{8} \qquad \frac{5}{16} + \frac{2}{16} = \frac{7}{16}$$

X2 X3

$$\frac{5}{9} + \frac{3}{6} \qquad \frac{10}{18} + \frac{9}{18} = \frac{19}{18} = 1\frac{1}{18}$$

X2 X2 X3

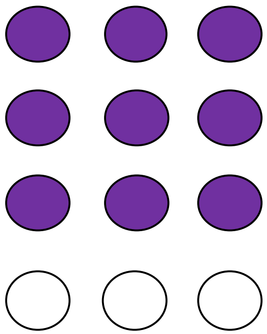
Before we can add these fractions, we must make sure they have a common denominator. To do this, we need to find the **lowest common multiple** of the denominators. The lowest common multiple is the lowest number that both numbers go into**.**

M5e: Can multiply fractions by whole numbers (fractions of quantities)

We can think of fractions in two ways: as a **number** and as an **operator.** When we place fractions on a number line and think of them as part of our number system we are thinking of fractions as **numbers.**

But fractions can also be used as **operators**. This requires us to use the fraction to carry out a calculation. Let's look at the example below:

We can find $\frac{3}{4}$ of a set of objects. Look at the visual example below:



$\frac{3}{4}$ of 12 is 9

In this example, the fraction $\frac{3}{4}$ is being used as an **operator** not a number because we have to find $\frac{3}{4}$ of a set of objects. We have to carry out an operation

To multiply a whole number by a fraction, you need to use the fraction as an operator. This means reading the multiplication sign as a fraction **of** a number.

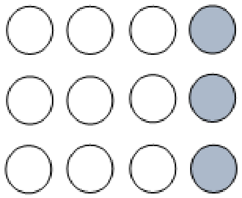
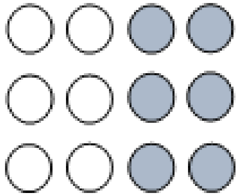
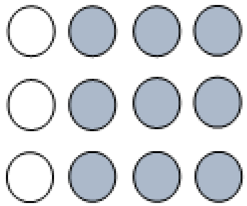
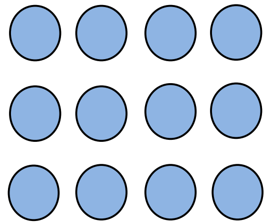
In the example left, $\frac{3}{4}$ **of** 12 is the same as $\frac{3}{4}$ **x** 12

How would you read $\frac{1}{4}$ x 12?

There are a number of ways we could think of $\frac{1}{4}$ x 12

12 lots of one quarter or one quarter 12 times

¼	**¼**	**¼**	**¼**	**¼**	**¼**
¼	**¼**	**¼**	**¼**	**¼**	**¼**

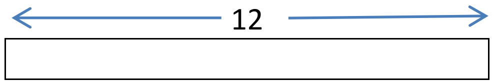
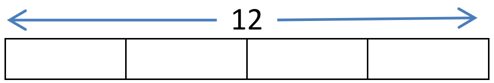
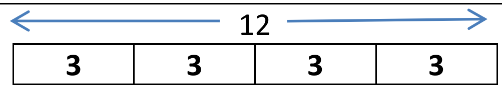
$\frac{1}{4}$ x 12 = $\frac{1}{4}$ of 12 = 3	$\frac{2}{4}$ x 12 = $\frac{2}{4}$ of 12 = 6	$\frac{3}{4}$ x 12 = $\frac{3}{4}$ of 12 = 9	$\frac{4}{4}$ x 12 = $\frac{4}{4}$ of 12 = 12
			

Remember: Finding $\frac{1}{4}$ of something is the same as dividing by 4

As a number sentence. For example, 12 grapes are shared equally between 4 friends. Each child will get:

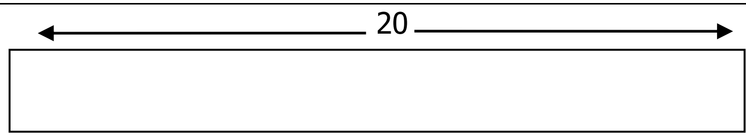
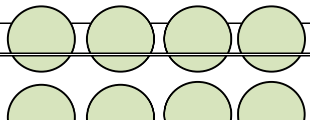
$\frac{1}{4}$ x 12 = 12 ÷ 4 = 3 grapes $\frac{1}{4}$ x 12 = 3

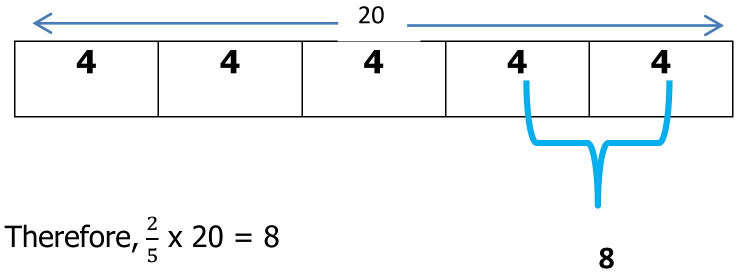
Using the bar model

		
First draw your bar	Next you need to find $\frac{1}{4}$ of the bar, so you need to divide it into 4 equal pieces	To find one piece you need to divide the whole bar into 12
The calculation is $\frac{1}{4}$ x 12, so the length of the bar is 12.	**Remember:** Finding $\frac{1}{4}$ of something is the same as dividing by 4	As there are 4 pieces, each piece is worth 3 (3 x 4 = 12)

So $\frac{1}{4}$ x 12 = 3

What about $\frac{2}{5}$ x 20?

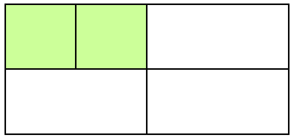
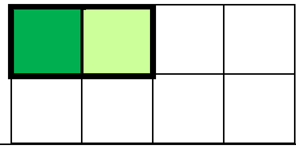
$\frac{2}{5}$ of 20 is 8	Using the Bar Method: $\frac{2}{5}$ x 20	
		

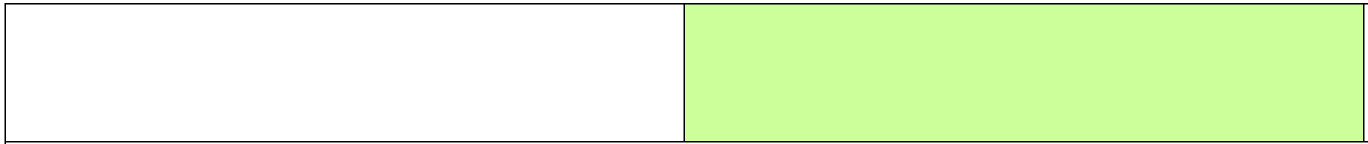
	To find $\frac{2}{5}$ you need to divide the bar into 5 pieces. We can see that dividing the bar into 5 means each section is 20 ÷ 5 = 4. If each section represents 4, then two sections is 2 x 4 = 8. Therefore, $\frac{2}{5}$ x 20 = 8 

M5f:	**Can multiply pairs of fractions, writing the answer in its simplest form**

Remember: Fractions can be used as numbers or operators. When multiplying two fractions, use one of the fractions as an operator. This means you are finding a fraction of a fraction	**Remember:** Finding $\frac{1}{4}$ of something is the same as dividing by 4.

$\frac{1}{2} \times \frac{1}{4}$	**This can be read as $\frac{1}{2}$ of a quarter, using one of the fractions as an operator.**

Draw a shape and divide it into quarters – four equal parts. 	**We need to find $\frac{1}{2}$ of one of the quarters.** **Finding $\frac{1}{2}$ of something is the same as dividing by 2** 	**To finish finding out the answer to $\frac{1}{2}$ of a quarter, we need to make all the parts in the diagram equal. This means we need to divide all the quarters into half.** 
	This shows one of the quarters divided into 2, or $\frac{1}{2}$ of a quarter.	**The part that is cross-shaded is ½ of a quarter.** **There are eight parts in total.**

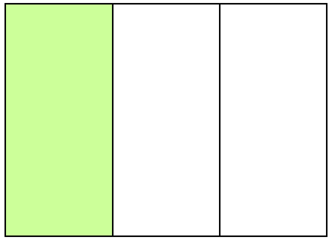


$\frac{1}{2}$ of a quarter is one eighth.

$$\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$$

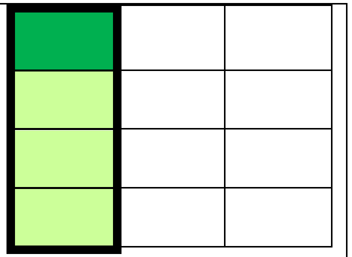
$\frac{1}{4} x \frac{1}{3}$	**This can be read as 1/4 of a third.**

First draw a diagram to show $\frac{1}{3}$



We know that finding $\frac{1}{4}$ is the same as dividing by 4.

So next we need to divide the third into 4.

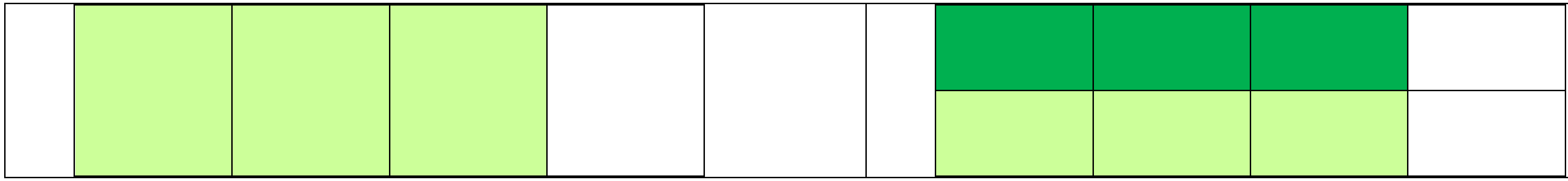


$$\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$$

This method can still be used when the numerator is greater than one.

$\frac{1}{2} x \frac{3}{4}$	**This can be read as $\frac{1}{2}$ of three quarters**

First draw a diagram to show $\frac{3}{4}$	$\frac{3}{4}$ divided by 2	



$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$

Written method

If you understand how to visually multiply fractions together, you have probably worked out the method for multiplying fractions together.

To multiply fractions the two numerators are multiplied together and the two denominators are multiplied together.

$\frac{1}{2} \times \frac{1}{4}$	$\frac{1 \times 1}{2 \times 4} = \frac{1}{8}$	$\frac{1}{5} \times \frac{1}{4}$	$\frac{1}{5} \times \frac{1}{4} = \frac{1}{20}$	$\frac{2}{3} \times \frac{1}{4}$	$\frac{2}{3} \times \frac{1}{4} = \frac{2}{12} = \frac{1}{6}$

M5g: Can divide fractions by whole numbers

Remember: Dividing by 4 is the same as finding $\frac{1}{4}$

Dividing a fraction by a whole number is the same as multiplying two fractions together.

This is because when you multiply two fractions together, one is being used as an operator.

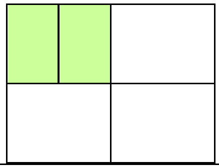
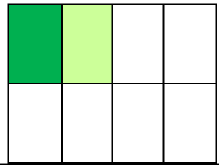
For example, $\frac{1}{2} \times \frac{1}{4}$ can be read as 'a half of a quarter'.

We know that finding a half of a number (in this case ¼) is the same as multiplying by ½.

Therefore, $\frac{1}{2} \times \frac{1}{4} = \frac{1}{4} \div 2$

Remember that when we are multiplying two numbers, we can work it out in any order.
So,

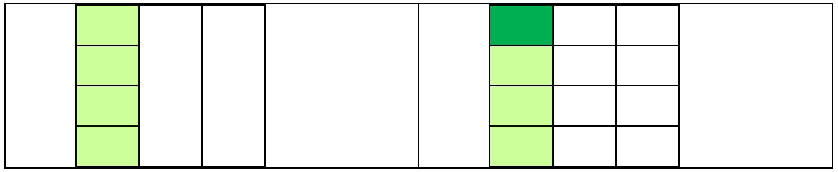
$\frac{1}{2} \times \frac{1}{4} = \frac{1}{4} \times \frac{1}{2}$

$\frac{1}{4} \div 2$	$\frac{1}{4} \div 2$ is **one eighth.**
	
$\frac{1}{3} \div 4 =$	$\frac{1}{3} \div 4 = \frac{1}{12}$

This can also be written as:

$\frac{1}{4} \times \frac{1}{2} = \frac{1}{2} \div 4$

Remember that finding ¼ of a number is the same as dividing the number by 4



Written method

We can divide a fraction by a whole number using our knowledge of multiplying fractions. Let's look at an example: $\frac{1}{2} \div 4$

Remember that dividing by 4 is the same as multiplying by ¼

Therefore, we can say that

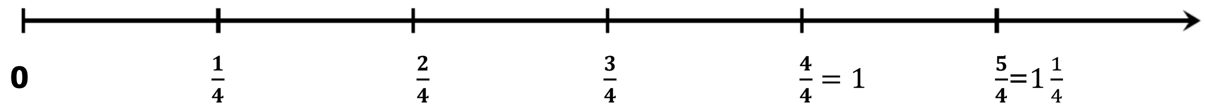
$\frac{1}{2} \div 4 = \frac{1}{2} \times \frac{1}{4}$ $\qquad$ $\frac{1}{2} \div 4 = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$

$\frac{1}{5} \div 6$	$\frac{1}{5} \div 6$ = $\frac{1}{5} \times \frac{1}{6}$ Equal to	$\frac{1}{5} \times \frac{1}{6} = \frac{1}{30}$ 1 x 1 = 1 / 2 x 6 = 30
$\frac{2}{3} \div 7$	$\frac{2}{3} \div 7$ = $\frac{2}{3} \times \frac{1}{7}$ Equal to	$\frac{2}{3} \times \frac{1}{7} = \frac{2}{21}$ 2 x 1 = 2 / 3 x 7 = 21

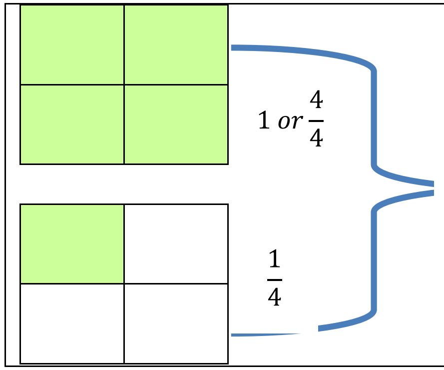
M5h: Can convert mixed numbers to improper fractions

Remember that fractions can be larger than 1

$1\frac{1}{4}$ is called a **mixed number**.	We read this as 'one **and** one quarter'.	The word '**and**' here is important because $1\frac{1}{4} = 1 + \frac{1}{4}$



$\frac{5}{4}$ is called an <u>improper fraction</u> or top-heavy fraction.	An improper fraction is a fraction where the **numerator** (top number) is larger than the **denominator** (bottom number).	$\frac{5}{4}$ is equal to $1\frac{1}{4}$.



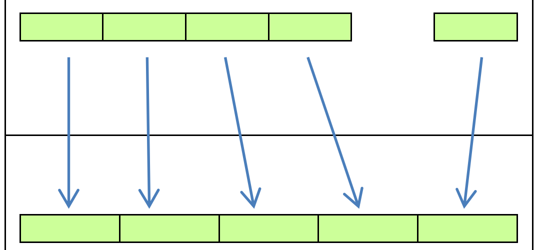
Mixed fraction

$$1\frac{1}{4} = \frac{5}{4}$$

Improper fraction

$$1\frac{1}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$$

$$\frac{5}{4} = +\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$$



How do we calculate without pictures?

$$2\frac{2}{3} = \frac{8}{3}$$

$$2 = \frac{6}{3} \quad + \quad \frac{2}{3}$$

so the calculation is $\frac{6}{3} + \frac{2}{3} = \frac{8}{3}$

Write $3\frac{2}{7}$ *as an improper fraction.*	**Start by writing 3 as sevenths** ✓ 1 whole one = 7 sevenths ✓ 2 whole ones = 14 sevenths ✓ 3 whole ones = 21 sevenths Therefore, **3** = $\frac{21}{7}$	**If 3 =** $\frac{21}{7}$**, then** $3\frac{2}{7} = \frac{21}{7} + \frac{2}{7} = \frac{23}{7}$ $3\frac{2}{7} = \frac{23}{7}$

M5j: Can read and write decimal numbers as fractions

Remember place value

In order to read and write decimal numbers as fractions, it is vital that you remember and understand the value of digits in numbers.

thousands	hundreds	tens	ones	.	tenths $\frac{1}{10}$	hundredths $\frac{1}{100}$	thousandths $\frac{1}{1000}$

The numbers get **smaller** as the chart moves to the **right**, so thousands are the biggest numbers and thousandths are the smallest numbers on this chart.

The chart mirrors itself in terms of the names of the columns with tens on the left of the decimal point and tenths on the right, then hundreds and hundredths etc.

Just as ten ones are 1 ten and 10 tens are 1 hundred, so 10 tenths are 1 one and 10 hundredths are 1 tenth.

If I want to read 0.1, I can put the number into the chart.

thousands	hundreds	tens	ones	.	tenths $\frac{1}{10}$	hundredths $\frac{1}{100}$	thousandths $\frac{1}{1000}$
			0	.	1		

So, 0.1 = $\mathbf{\frac{1}{10}}$

0.4	=	four tenths	=	$\frac{4}{10} = \frac{2}{5}$		0.7	=	seven tenths	=	7/10
0.5	=	five tenths	=	$\frac{5}{10} = \frac{1}{2}$		0.8	=	eight tenths	=	8/10 = 4/5
0.6	=	six tenths	=	$\frac{6}{10} = \frac{3}{5}$		0.9	=	nine tenths	=	9/10

What about 0.01

thousands	hundreds	tens	ones	.	tenths	hundredths	thousandths
			0	.	0	1	

In the chart, we can see that this number can be read as one hundredth. This can be written as 1/100.

0.01 $= \frac{1}{100}$

thousands	hundreds	tens	ones	.	tenths	hundredths	thousandths
			0	.	0	2	

This is two hundredths or $\frac{2}{100}$.

0.02 = $\frac{2}{100} = \frac{1}{50}$

0.03 = three hundredths = $\frac{3}{100}$

0.04 = four hundredths = $\frac{4}{100} = \frac{1}{25}$

0.05 = five hundredths = $\frac{5}{100}$ = $\frac{1}{20}$

What is 0.37 as a fraction?	***What is 0.45 as a fraction?***
If we use the place value chart again, we can see the 0.37 is 3 tenths and 7 hundredths. 0.37 = $\frac{3}{10} + \frac{7}{100}$ We know that $\frac{3}{10} = \frac{30}{100}$ Therefore, 0.37 = $\frac{30}{100} + \frac{7}{100} = \frac{37}{100}$	Here, 0.45 = $\frac{45}{100}$ In this example though, $\frac{45}{100}$ is not a fraction in its simplest form because 45 and 100 have a **common factor** of **5**. Divide both 45 and 100 by 5 $\frac{45}{100} = \frac{9}{20}$ Therefore, 0.45 = $\frac{45}{100} = \frac{9}{20}$

We can keep counting up in hundredths. There are some well known decimal hundredths that convert to fractions.

0.10 = ten hundredths = $\frac{10}{100}$ = $\frac{1}{10}$

0.25 = $\frac{25}{100}$ = $\frac{1}{4}$ (one quarter)

0.75 = $\frac{75}{100}$ = $\frac{3}{4}$ (three quarters)

What about 0.001?

thousands	hundreds	tens	ones	.	tenths	hundredths	thousandths
			0		0	0	1

Once written in the chart, this can easily be read as one thousandth or $\frac{1}{1000}$.

0.01 = $\frac{1}{1000}$

thousands	hundreds	tens	ones	.	tenths	hundredths	thousandths
			0		0	0	2

0.02 = two thousandths = $\frac{2}{1000}$ = $\frac{1}{500}$

What is 0.381 as a fraction?

$$0.381 = \frac{3}{10} + \frac{8}{100} + \frac{1}{1000} = \frac{300}{1000} + \frac{80}{1000} + \frac{1}{1000} = \mathbf{\frac{381}{1000}}$$

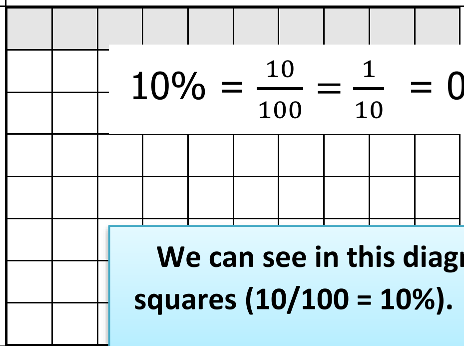
0.125 is one hundred and twenty five thousandths or $\frac{125}{1000}$. This can be reduced to $\frac{25}{200}$ or $\frac{5}{40}$ or $\frac{1}{8}$.

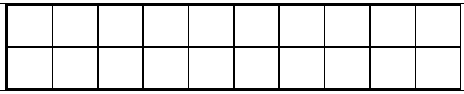
0.333 is approximately $\frac{1}{3}$.

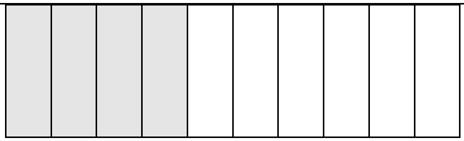
0.667 is approximately two thirds or $\frac{2}{3}$.

M5k:	**Can recognise approximate proportions of a whole number using percentages**

Remember: A percentage represents the number of parts out of 100. For example, 76% means 76 out of 100 and 100% means 100 out of 100 (or a whole). Percentages can also be written as fractions and decimals.

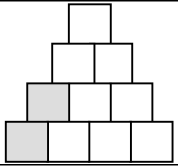
Equivalence	$50\% = \frac{50}{100} = \frac{1}{2} = 0.5$ $25\% = \frac{25}{100} = \frac{1}{4} = 0.25$ $75\% = \frac{75}{100} = \frac{3}{4} = 0.75$
Number line	0 25% 50% 75% 100% $\frac{25}{100} = \frac{1}{4}$ $\frac{50}{100} = \frac{1}{2}$ $\frac{75}{100} = \frac{3}{4}$
Pictorial	 $10\% = \frac{10}{100} = \frac{1}{10} = 0.1$ **We can see in this diagram that 10 squares are shaded out of a total of 100 squares (10/100 = 10%). We can also see that 1 row is shaded out of a total of 10 rows (1/10 = 10%)**





4 out of 10 blocks are shaded

$\frac{4}{10} = \frac{40}{100} =$ **40%**



2 out of 10 blocks are shaded.

$\frac{2}{10} = \frac{20}{100} =$ **20%**

- ✓ Count how many squares there are in total- this is your denominator
- ✓ Count how many squares are shaded- this is your numerator
- ✓ Look at the fraction- is there an equivalent that would support your calculation?
- ✓ Convert to a decimal or percentage- what is the question asking you?

Using known equivalent facts to help you calculate percentages. **You must learn the following facts:**

10% = $\frac{1}{10}$

20% = $\frac{2}{10} = \frac{1}{5}$

25% = $\frac{25}{100} = \frac{1}{4}$

50% = $\frac{50}{100} = \frac{1}{2}$

75% = $\frac{75}{100} = \frac{3}{4}$

Once you know these percentages and their fraction equivalents, you can find others.

For example,

10% = $\frac{1}{10}$

From this we can work out that:

40% = $\frac{4}{10}$

or 60% = $\frac{6}{10}$

We know that $\frac{1}{5}$ = 20%

We therefore can work out that:

$\frac{2}{5}$ = 40%

$\frac{3}{5}$ = 60%

$\frac{4}{5}$ = 80%

Often we are asked to find percentages of certain numbers. Sometimes, the easiest way to do that is convert the percentage to a fraction and then work it out. Let's look at some examples:

25% of 12

We know that 25% = $\frac{1}{4}$

$\frac{1}{4}$ of 12 = 3 Therefore, 25% of 12 = 3

50% of 32

We know that 50% = $\frac{1}{2}$

$\frac{1}{2}$ of 32 = 16

Calculating percentages of an amount using known facts:

15% of 38= 15% =10% + 5%

First we need to find 10% by dividing the number by 10

38 ÷10 = 3.8

10% = 3.8

35% of 125= 35% = 10% + 10% + 10% + 5%

first we need to find 10% by dividing the number by 10

125 ÷10 = 12.5

10% = 12.5

30% = 12.5 x 3 = or 12.5 + 12. 5 + 12.5 =

If we know 10% we can now find 5%
as half of 10 is 5 we can half 10%

10%	=	3	.	8
5%	=	1	.	9
15%	=	5	.	7

1

10% = 3.8

5% = Half of 3 = 1.5 **+** Half of 0.9= 0.4

5% =1.9

30% = 37.5

30%	=	3	7	.	5	0
5%	=		6	.	2	5
15%	=	4	3	.	7	5

1

If we know 10% we can now find 5%
as half of 10 is 5 we can half 10%

10% = 12.5

5% = Half of 12 = 6 **+** Half of 0.5= 0.25

5%= 6.25

M5l:	Can recognise simple equivalence between fractions, decimals and percentages

To recognise simple equivalences between fraction, decimals and percentages we must first remember that equivalent means equal to.

Fractions, decimals and percentages are all ways of representing parts of a whole.

Fractions	Decimals	Percentages	Fractions	Decimals	Percentages
$^{1}/_{2}$	0.50	50%	$^{1}/_{6}$	0.16	16 $^{2}/_{3}$%
$^{1}/_{3}$	$0.3\dot{3}$	33 $^{1}/_{3}$	$^{1}/_{6}$	0.125	12 $^{1}/_{2}$%
$^{2}/_{3}$	$0.6\dot{6}$	66 $^{2}/_{3}$	$^{3}/_{8}$	0.375	37 $^{1}/_{2}$%
$^{1}/_{4}$	0.25	25%	$^{5}/_{8}$	0.625	62 $^{1}/_{2}$%
$^{3}/_{4}$	0.75	75%	$^{7}/_{8}$	0.875	87$^{1}/_{2}$%
$^{1}/_{5}$	0.20	20%	$^{1}/_{10}$	0.10	10%
$^{2}/_{5}$	0.40	40%	$^{3}/_{10}$	0.30	30%
$^{3}/_{5}$	0.60	60%	$^{5}/_{10}$	0.5	50%

$^{4}/_{5}$	0.80	80%		$^{9}/_{10}$	0.9	90%